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DETERMINATION OF INTERNAL HEAT AND MASS TRANSFER PARAMETERS USING CHARACTERISTIC FUNCTIONS OF THE THERMODYNAMICS OF IRREVERSIBLE PROCESSES

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Temperature and moisture content measurements are used to obtain experimental variables which are arguments of characteristic functions of the Massieu type in the thermodynamics of irreversible processes. By means of these variables the thermal diffusivity and dimensionless inertia, thermal diffusion and phase transition parameters are calculated.

Determination of internal transfer parameters from multi-point temperature measurements. It has been shown [1] that from measurements of the temperature $t(N_0, \tau)$ at a point N_0 in a body and of its mean moisture content, which are experimental functions of time, the fields of heat and mass content can be reconstructed, i.e., they may be expressed in terms of the experimental functions

$$t(N, H) = \sum_{n=0} t_0^{(n)} (H) P_n (N) + u_v^{(n)} (H) Q_n (N), \tag{1}$$

$$u(N, H) = \sum_{n=0} t_0^{(n)} (H) S_n (N) + u_v^{(n)} (H) V_n (N). \tag{2}$$

In an actual experiment these sums are always finite, since the errors in the derivatives increase in geometric progression with increase of their order. If the temperature is measured during the experiment at the (r + 1)-th point of the body N_j, then, expressing temperature $t(N_{i+1}, \tau)$ in terms of the temperature derivatives at point N_i, we may denote r by the finite sum

$$\Delta (N_i, N_{i+1}, \tau) = t(N_{i+1}, \tau) - t(N_i, \tau) = \\ = \sum_{p=1}^{n} t_i^{(p)} (\tau) \left(\frac{R^2}{a}\right)^p P_p (N) + \sum_{q=1}^{m} u_v^{(q)}(\tau) \left(\frac{R^2}{a}\right)^q Q_q (N), \tag{3}$$

the number of terms of which depends on the number n of derivatives of temperature $t_i(\tau)$ and on the number m of derivatives of mean moisture content $u_v(\tau)$, which are appreciably larger than their errors, i.e., reliable. The system (3) cannot be used to calculate transfer parameters when the number of reliable derivatives of moisture content $m > 1$, since its determinant is then zero. Therefore, for multi-point temperature measurements, we must either take time intervals small enough for the drying rate $u'_v(\tau)$ to be considered constant and $u''_v(\tau)$ to be neglected in each interval, or we must use temperature measurements at successive times and allow for variation of $u'_v(\tau)$ and its derivatives. The first method is convenient for calculating the thermal diffusivity a and the complex ε Ko, and the second for calculating Lu and Pn.

Determination of transfer parameters from two-point temperature measurements. The finite sum containing reliable derivatives of the measured temperature $t(N_0, \tau) = t_0(\tau)$ and moisture content $u_v(\tau)$

$$\Delta (N_0, N, \tau) = \sum_{p=1}^{n} t_0^{(p)} (\tau) P_p (N) \left(\frac{R^2}{a}\right)^p + \sum_{q=1}^{m} u_v^{(q)} (\tau) Q_q (N) \left(\frac{R^2}{a}\right)^q \tag{4}$$

permits one to find the internal transfer parameters, if one uses its values at successive times

$$\tau_1 < \tau_2 < \ldots < \tau_r; \quad r = n + m. \tag{5}$$

We shall designate products of radial polynomials and powers of R^2/a

$$\xi_p = (R^2 a^{-1})^p P_p (N); \quad \eta_q = (R^2 a^{-1}) Q_q (N), \tag{6}$$

and then the sum (4), written for moments of time τ_s, forms a set of equations from which ξ_p and η_q may be found. These unknowns contain the dimensionless internal transfer numbers ε Ko, Lu, Pn and the geometric parameter $R^2 a^{-1}$.

To determine the four parameters named, a set of four equations is sufficient. We can set up $C_r^4 = r!/(r-4)!4!$ such systems from the ensemble (4), each permitting a calculation of its approximation to the transfer parameters. For the calculations it is necessary to know the determinant of system (4)

$$D = \begin{vmatrix} t_0'(\tau_1) & \ldots & t_0^{(n)}(\tau_1) & u_v'(\tau_1) & \ldots & u^{(m)}(\tau_1) \\ \cdot & \ldots & \cdot & \cdot & \ldots & \cdot \\ \cdot & \ldots & \cdot & \cdot & \ldots & \cdot \\ \cdot & \ldots & \cdot & \cdot & \ldots & \cdot \\ t_0'(\tau_r) & \ldots & t_0^{(n)}(\tau_r) & u_v'(\tau_r) & \ldots & u^{(m)}(\tau_r) \end{vmatrix} \tag{7}$$

and the determinants D_s obtained from this determinant by replacing column S with the experimental temperature differences $\Delta(N_0\, N_i,\, \tau) = t(N_i,\, \tau) - t(N_0,\, \tau)$.

The thermal diffusivity is given by the ratio

$$a = P_1(N)\, R^2 D/D_1, \tag{8}$$

the complex dimensionless number for which is

$$\varepsilon\,\mathrm{Ko} = -D_{n+1}/D_1, \tag{9}$$

the thermal diffusion number

$$\mathrm{Pn} = \frac{2(k+2)(N^2 - N_0^2)\, DD_2 - [(k+4)\, N_0^2 - kN^2]\, D_1^2}{k\,(2 - N^2 - N_0^2)\, D_1 D_{n+1}}, \tag{10}$$

and the inertia simplex

$$\mathrm{Lu}^{-1} = \frac{2(k+2)}{k\,(2 - N^2 - N_0^2)}\, \frac{D}{D_1} \left[\frac{D_2}{D_1} - \frac{D_{n+2}}{D_{n+1}} \right]. \tag{11}$$

Measurements of temperature and moisture content may also be less accurate, allowing calculation only of their first derivatives. In this kind of linear approximation the temperature field is described by only three components

$$\Delta(N_0,\, N,\, \tau) = [t_0'(\tau) - \varepsilon\,\mathrm{Ko}\, u_v'(\tau)]\, \frac{N^2 - N_0^2}{2k}\, \frac{R^2}{a}, \tag{12}$$

and it is quite insufficient for finding Lu and Pn. However, the diffusivity and the complex number may be calculated even from such measurements

$$\begin{gathered} a = \frac{N_1^2 - N_0^2}{2k}\, R^2 \begin{vmatrix} t'(N_0,\, \tau_1) & u_v'(\tau_1) \\ t'(N_0,\, \tau_2) & u_v'(\tau_2) \end{vmatrix} : \begin{vmatrix} \Delta(N_0,\, N_1,\, \tau_1) & u_v'(\tau_1) \\ \Delta(N_0,\, N_1,\, \tau_2) & u_v'(\tau_2) \end{vmatrix}, \\ \varepsilon\,\mathrm{Ko} = \begin{vmatrix} \Delta(N_0,\, N_1,\, \tau_1) & t'(N_0,\, \tau_1) \\ \Delta(N_0,\, N_1,\, \tau_2) & t'(N_0,\, \tau_2) \end{vmatrix} : \begin{vmatrix} \Delta(N_0,\, N_1,\, \tau_1) & u_v'(\tau_1) \\ \Delta(N_0,\, N_1,\, \tau_2) & u_v'(\tau_2) \end{vmatrix}. \end{gathered} \tag{13}$$

Determination of transfer criteria using local moisture content measurements. The internal transfer parameters may be found with the aid of local moisture content measurements, as follows from sum (2) and the expressions for the radial polynomials $S_p(N)$ and $V_q(N)$, given in [1]. The corresponding system of equations is obtained from the sum

$$u(N,\, \tau) - u_v(\tau) = \sum_{p=1}^{n} t_0^{(p)}(\tau)\, S_p(N) \left(\frac{R^2}{a}\right)^p + \sum_{q=1}^{m} u_v^{(q)}(\tau)\, V_q(N) \left(\frac{R^2}{a}\right)^q, \tag{14}$$

referred to times $\tau_1 < \tau_2 < \ldots < \tau_r$. In the crudest approximation the moisture content field is approximated by the first derivatives of local temperature $t_0(\tau)$ and of mean moisture content $u_v(\tau)$

$$u(N,\, \tau) - u_v(\tau) = \frac{1}{2}\left[\frac{1}{k+2} - \frac{N^2}{k}\right] \frac{R^2}{a} \times$$

$$\times\, [\mathrm{Pn}\, t_0'(\tau) - (\varepsilon\,\mathrm{Ko}\,\mathrm{Pn} + \mathrm{Lu}^{-1})\, u_v'(\tau)]. \tag{15}$$

If the diffusivity a and the complex ε Ko are calculated previously from temperature measurements, then from the system

$$\omega(N_0, N_1, N_2, \tau_s) = -t'(N_0, \tau_s)\,\mathrm{Pn} + u_v'(\tau_s)(\varepsilon\,\mathrm{Ko}\,\mathrm{Pn} + \mathrm{Lu}^{-1}), \tag{16}$$

in which the difference

$$\omega(N_0, N_1, N_2, \tau) = \frac{2k(k+2)a}{[(k+2)N_2^2 - k]R^2}[u(N_2, \tau) - u_v(\tau)], \tag{17}$$

we may find the thermal diffusion number

$$\mathrm{Pn} = -\begin{vmatrix} \omega(N_0, N_1, N_2, \tau_1) & u_v'(\tau_1) \\ \omega(N_0, N_1, N_2, \tau_2) & u_v'(\tau_2) \end{vmatrix} : \begin{vmatrix} t_0'(\tau_1) & u_v'(\tau_1) \\ t_0'(\tau_2) & u_v'(\tau_2) \end{vmatrix} \tag{18}$$

and the inertia number

$$\mathrm{Lu} = \begin{vmatrix} t_0'(\tau_1) & u_v'(\tau_1) \\ t_0'(\tau_2) & u_v'(\tau_2) \end{vmatrix} : \begin{vmatrix} t_0'(\tau_1) - \varepsilon\,\mathrm{Ko}\,u_v'(\tau_1) & \omega(N_0, N_1, N_2, \tau_1) \\ t_0'(\tau_2) - \varepsilon\,\mathrm{Ko}\,u_v'(\tau_2) & \omega(N_0, N_1, N_2, \tau_2) \end{vmatrix}. \tag{19}$$

In the stage of falling drying rate the moisture content at points on the body surface may be considered equal to zero. Then

$$\omega(N_0, N_1, 1, \tau) = -\frac{k(k+2)a}{R^2}u_v(\tau). \tag{20}$$

The complex number

$$\ni = \mathrm{Lu}^{-1} + \varepsilon\,\mathrm{Ko}\,\mathrm{Pn}, \tag{21}$$

which is also obtained by solving (16), is the ratio of the determinants

$$\ni = \begin{vmatrix} t_0'(\tau_1) & \omega(N_0, N_1, N_2, \tau_1) \\ t_0'(\tau_2) & \omega(N_0, N_1, N_2, \tau_2) \end{vmatrix} : \begin{vmatrix} t_0'(\tau_1) & u_v'(\tau_1) \\ t_0'(\tau_2) & u_v'(\tau_2) \end{vmatrix}. \tag{22}$$

Internal transfer parameters as characteristic functions of the thermodynamics of irreversible processes. The internal transfer parameters are represented in the following formulas as the ratio of determinants of the second order

$$\Pi = \begin{vmatrix} x(\tau_1) & y(\tau_1) \\ x(\tau_2) & y(\tau_2) \end{vmatrix} : \begin{vmatrix} \xi(\tau_1) & \eta(\tau_1) \\ \xi(\tau_2) & \eta(\tau_2) \end{vmatrix}, \tag{23}$$

in which the experimental quantities $t'_0(\tau)$, $u'_v(\tau)$, Δ, w, $\omega = x$, y, ξ and η are taken at such times τ_1 and τ_2, when these determinants differ from zero, i.e., they are much greater than their errors. The transfer parameters must not be determined by the method described at constant drying rate or near its maximum, when all the above determinants vanish because of the steady temperature field. Times $\tau_1 = \tau$ and τ_2 can always be taken sufficiently close to $\tau = \tau + d\tau$. According to Lagrange's mean theorem $f(\tau + d\tau) = f(\tau) + f'(\tau)d\tau$, relation (4) goes over to the ratio of the two Wronskians

$$\Pi = \begin{vmatrix} x(\tau) & y(\tau) \\ x'(\tau) & y'(\tau) \end{vmatrix} : \begin{vmatrix} \xi(\tau) & \eta(\tau) \\ \xi'(\tau) & \eta'(\tau) \end{vmatrix}. \tag{24}$$

Thus, an expression for the diffusivity may be obtained from (13)

$$a = \frac{N^2 - N_1^2}{2k}R^2 \begin{vmatrix} t'(N_0, \tau) & u_v'(\tau) \\ t''(N_0, \tau) & u_v''(\tau) \end{vmatrix} : \begin{vmatrix} \Delta(N_0, N_1, \tau) & u_v'(\tau) \\ \Delta'(N_0, N_1, \tau) & u_v''(\tau) \end{vmatrix}, \tag{25}$$

and similar expressions for all the other internal transfer numbers may be found from (13), (18), (19) and (22):

$$\varepsilon\,\mathrm{Ko} = \begin{vmatrix} \Delta(N_0, N_1, \tau) & t'(N_0, \tau) \\ \Delta'(N_0, N_1, \tau) & t''(N_0, \tau) \end{vmatrix} : \begin{vmatrix} \Delta(N_0, N_1, \tau) & u_v'(\tau) \\ \Delta'(N_0, N_1, \tau) & u_v''(\tau) \end{vmatrix}, \tag{26}$$

$$\mathrm{Pn} = -\begin{vmatrix} \omega(N_0, N_1, N_2, \tau) & u_v'(\tau) \\ \omega'(N_0, N_1, N_2, \tau) & u_v''(\tau) \end{vmatrix} : \begin{vmatrix} t'(N_0, \tau) & u_v'(\tau) \\ t''(N_0, \tau) & u_v''(\tau) \end{vmatrix}, \tag{27}$$

$$\mathrm{Lu} = \begin{vmatrix} t'(N_0, \tau) & u_v'(\tau) \\ t''(N_0, \tau) & u_v''(\tau) \end{vmatrix} : \begin{vmatrix} t'(N_0, \tau) - \varepsilon\,\mathrm{Ko}\, u_v'(\tau) & \omega(N_0, N_1, N_2, \tau) \\ t''(N_0, \tau) - [\varepsilon\,\mathrm{Ko}\, u_v'(\tau)]' & \omega'(N_0, N_1, N_2, \tau) \end{vmatrix}, \tag{28}$$

$$Э = \begin{vmatrix} t'(N_0, \tau) & \omega(N_0, N_1, N_2, \tau) \\ t''(N_0, \tau) & \omega'(N_0, N_1, N_2, \tau) \end{vmatrix} : \begin{vmatrix} t'(N_0, \tau) & u_v'(\tau) \\ t''(N_0, \tau) & u_v''(\tau) \end{vmatrix}, \tag{29}$$

where throughout the prime denotes differentiation with respect to time.

For any of the transfer parameters (24)–(29) four equivalent forms may be written:

$$\begin{aligned} \Pi &= \left(\frac{x}{\xi}\right)^2 \left[\frac{y}{x}\right]' \Big/ \left[\frac{\eta}{\xi}\right]' = \left(\frac{y}{\eta}\right)^2 \left[\frac{x}{y}\right]' \Big/ \left[\frac{\xi}{\eta}\right]' = \\ &= -\left(\frac{x}{\eta}\right)^2 \left[\frac{y}{x}\right]' \Big/ \left[\frac{\xi}{\eta}\right]' = -\left(\frac{y}{\xi}\right)^2 \left[\frac{x}{y}\right]' \Big/ \left[\frac{\eta}{\xi}\right]', \end{aligned} \tag{30}$$

of which we may always choose the simplest. For this purpose new arguments must be introduced:

$$Z = y/x; \quad \zeta = \eta/\xi, \tag{31}$$

in the functions of which the transfer parameters are expressed either as derivatives or as characteristic thermodynamic functions of irreversible transfer processes of Massieu-Gibbs-Helmholtz type [2–4].

For diffusivity the experimentally observed functions of time are

$$\begin{aligned} x_a &= u_v'(\tau), & y_a &= t'(N_0, \tau), \\ \xi_a &= u_v'(\tau), & \eta_a &= \Delta(N_0, N_1, \tau). \end{aligned} \tag{32}$$

The arguments corresponding to the functions are

$$Z_a = t'(N_0, \tau)/u_v'(\tau); \quad \zeta_a = \Delta(N_0, N_1, \tau)/u_v'(\tau), \tag{33}$$

for which the diffusivity proves to be proportional to the derivative

$$a = \frac{N^2 - N_0^2}{2k} R^2 \left[\frac{\partial Z_a}{\partial \zeta_a}\right]_{N_0,\, N_1 = \mathrm{const}}. \tag{34}$$

The complex ε Ko (13), (22) is calculated from the experimental values

$$\begin{aligned} x_E &= \Delta(N_0, N_1, \tau), & y_E &= t'(N_0, \tau), \\ \xi_E &= \Delta(N_0, N_1, \tau), & \eta_E &= u_v'(\tau). \end{aligned} \tag{35}$$

The subsidiary arguments

$$Z_E = t'(N_0, \tau)/\Delta(N_0, N_1, \tau); \quad \zeta_E = u_v'(\tau)/\Delta(N_0, N_1, \tau) \tag{36}$$

allow us to represent this complex as the derivative

$$\varepsilon\,\mathrm{Ko} = \left[\frac{\partial Z_E}{\partial \zeta_E}\right]_{N_0,\;N_1=\mathrm{const}} . \qquad (37)$$

It should be borne in mind, however, that the arguments Z_E and ζ_E of the complex ε Ko are intimately connected with the diffusivity arguments

$$Z_E = Z_a/\zeta_a, \quad \zeta_E = 1/\zeta_a, \qquad (38)$$

and in its dependence on these variables the complex number ε Ko is a characteristic function of the Massieu-Gibbs type:

$$\varepsilon\,\mathrm{Ko} = Z_a - \zeta_a \frac{\partial Z_a}{\partial \zeta_a} . \qquad (39)$$

Experimental functions of the inertia simplex Lu. The thermal gradient number (18), (27) is found from the experimental functions

$$x_p = u_v'(\tau), \qquad y_p = \omega(N_0,\ N_1,\ N_2,\ \tau),$$
$$\xi_p = u_v'(\tau), \quad \eta_p = t'(N_0,\ \tau), \qquad (40)$$

to which correspond the arguments

$$Z_p = \omega(N_0,\ N_1,\ N_2,\ \tau)/u_v'(\tau), \quad \zeta_p = t'(N_0,\ \tau)/u_v'(\tau) = Z_a. \qquad (41)$$

The thermal gradient number expressed in terms of these functions

$$\mathrm{Pn} = -[\partial Z_p/\partial \zeta_p] = -[\partial Z_p/\partial Z_a] \qquad (42)$$

is also written as a derivative.

The inertia simplex (19), (28) has its experimental functions

$$x_L = \omega(N_0,\ N_1,\ N_2,\ \tau), \ y_L = t'(N_0,\ \tau) - \varepsilon\,\mathrm{Ko}\, u_v'(\tau),$$
$$\xi_L = u_v'(\tau), \qquad \eta_L = t'(N_0,\ \tau), \qquad (42a)$$

which form arguments

$$Z_L = \frac{\zeta_a}{Z_p} \frac{\partial Z_a}{\partial \zeta_a}, \ \zeta_L = Z_a, \qquad (43)$$

depending as follows, on the arguments of the foregoing transfer parameters:

$$Z_L = \frac{\zeta_a}{Z_p} \frac{\partial Z_a}{\partial \zeta_a}, \quad \zeta_L = Z_a. \qquad (43a)$$

From the general law (30) the inertia simplex

$$\mathrm{Lu}^{-1} = Z_p^2 \frac{\partial Z_L}{\partial \zeta_L} \qquad (44)$$

is proportional to the derivative of its arguments. Substitution of (43a) and allowance for the fact that

$$\frac{\partial}{\partial Z_a}\left(\frac{\partial Z_a}{\partial \zeta_a}\right) = 0$$

leads to a characteristic equation of Massieu-Gibbs type for the complex Lu

$$\mathrm{Lu}^{-1} = Z_p - \zeta_a \frac{\partial Z_p}{\partial \zeta_a} . \qquad (45)$$

The complex number (22), (29) is calculated from the experimental functions

$$x_{\text{э}} = t'(N_0,\ \tau),\quad y_{\text{э}} = \omega(N_0,\ N_1,\ N_2,\ \tau),$$
$$\xi_{\text{э}} = t'(N_0,\ \tau),\ \ \eta_{\text{э}} = u_v'(\tau), \tag{46}$$

which form the subsidiary arguments

$$Z_{\text{э}} = Z_p/Z_a,\qquad \zeta_{\text{э}} = 1/Z_a. \tag{47}$$

In the functions of these arguments the complex Э is expressed by an equation of the Massieu-Gibbs-Helmholtz type of the thermodynamics of irreversible processes

$$\text{Э} = Z_p - Z_a\,\frac{\partial Z_p}{\partial Z_a}\ . \tag{48}$$

Clearly, the number Pn is expressed in linear-fractional fashion in terms of the characteristic functions ε Ko, Lu and Э, the arguments Z_a, ζ_a and Z_p of which, in contrast to the arguments of the characteristic functions of classical thermodynamics, necessarily depend on the time τ. Thus the thermodynamic method of investigation is applicable to unsteady heat and mass transfer processes.

Calculation of transfer parameters as Massieu type characteristics of the thermodynamics of irreversible processes. Internal transfer parameters have been calculated from experiment by M. V. Popov [5, 6]. A ceramic plate of thickness 2R = 51 mm was used, the initial moisture content being 375 kg/m^3, porosity 38%, and γ_0 = 1464 kg/m^3. The temperature of the metal radiators was 250° C. A thermocouple located on the surface was not used for the calculation, since it was irradiated by the radiator. For the calculations a point 2.5 mm away from the surface was chosen, its dimensionless coordinate being N_1 = 0.902, and the second point was taken at the center N_0 = 0. The initial temperature of the substance was 15° C, the final 95° C, and the numbers Pn and ε Ko were referred to a temperature difference of 80° C. Then the dimensionless temperature

$$t = \frac{\tilde{t}(x,\ \tau) - 15^\circ\,\text{C}}{95^\circ\,\text{C} - 15^\circ\,\text{C}}\ ; \tag{49}$$

dimensioness time was referred to 500 min:

$$\tau = \vartheta\,\text{min.}/500\ \text{min.}; \tag{50}$$

the dimensionless interval h = 0.04 corresponded to a time of 20 min. Differentiation of temperature was carried out according to (7), and for the end (τ = 0 and 31) and corner (τ = 16.3; 18 and 26.7) points "limit" formulas were used, permitting calculation of left and right derivatives with respect to three experimental temperatures. An intermediate method allowing determination of derivatives with respect to four experimental values proved to be suitable for neighboring points. The drying curve did not have singularities, and the drying rate was found by differentiation in an interval containing 5, 7, and 9 experimental values of the moisture content. Differentiation of mass content when its values were small brought in large errors, since the change of moisture center at the end of the process was comparable with the weighting error. For this reason the transfer parameters were not calculated at the point τ = 30.

Table 1 shows the experimental values of the experimental functions and their derivatives with respect to time τ. It may be seen from the graph of [5, 6] that the evaporation surface at time τ_1 = 18 passes through the point N_1 = 0.902, and at time τ_2 = 26.7 it reaches the center of the body N_0 = 0. The corner points of the temperature curves correspond to these times. In the interval (7; 13) there is a very diffuse maximum of drying rate, in the vicinity of which the arguments Z_a and ζ_a are practically constant, and the derivative $\partial Z_a/\partial\zeta_a$ cannot be evaluated. For this reason the period $\tau \in$ (8; 11) of constant drying rate is not used for calculating the transfer parameters and is omitted from Table 2. The derivatives of arguments Z_a and ζ_a in the increasing drying rate stage $\tau \in$ (1; 7) and in the falling drying rate stage $\tau \in$ (12; 18) were calculated from seven points [7, 8], and at the end of the process from the last three $\tau \in$ (27; 29).

The phenomenological system of heat and mass transfer equations was described in [9] for zonal calculation of the process, i.e., it was applied to points that are not divided by a surface of phase transition, where the bond between the moisture and the substance is the same. It was applied in just this way in references [10–16, 18, 19, etc.]. From time τ_1 = 18 even up to τ_2 = 26.7, the body center N_0 = 0 and the point N_1 = 0.902 were separated by the evaporation surface studied. Therefore in the time range $\tau \in (\tau_1;\ \tau_2)$ there are no expansions of (1) and (2), since they were obtained by solving a system in which movement of the phase transition surface was not taken into account. Naturally, all the effects of these expansions—relations of the Massieu-Gibbs type—cannot be used in the time interval $(\tau_1;\tau_2)$. The inequality

$$\partial Z_a / \partial \zeta_a < 0 \tag{51}$$

corresponds to such a location of the evaporation surface between points for which arguments Z_a and ζ_a were constructed. The Massieu-Gibbs relations for the transfer parameters occur until time τ_1 and after time τ_2, when both points N_0 and N_1 are located in one zone, and the expansions of (1) and (2) are valid. Until the evaporation surface, passes through a point, the moisture travels along the capillaries as a connected liquid, but after the surface passes it becomes a disconnected system of threads and migrates along the capillaries solely as vapor. It can be seen from Table 2 that after time $\tau = 2$ the complex ε Ko increases monotonically, and becomes steady near the value 1.09 in the stage of constant drying rate and at the start of falling drying rate, i.e., until the main mass of liquid breaks up into threads. After the moisture has passed into the disconnected state, the complex ε Ko increases sharply, this being due to increase of ε and decrease of the heat capacity of the moist material c = (0.21 + 0.256), as well as possible increase of ρ. The comparatively large value of ε Ko at the very beginning of the process may evidently be explained by additional absorption of heat, which is expended in breaking down the surface film of the moist substance and creating a capillary surface of evaporation. This area may be calculated from the value of $\varepsilon \rho$ and the forces of surface tension.

Table 1

Experimental Values and Their Derivatives.

25τ	$u_v(\tau)$	$t(N_0, \tau)$	$t(N_1, \tau)$	Δ	$u_v'(\tau)$	$t'(N_0\ \tau)$
0	1.000	0.0000	0.000	0.000	—0.287	5.412
1	0.980	0,186	0.266	0.080	—0.705	3.854
2	0.943	0.311	0.349	0.038	—1,007	2.215
3	0.895	0.348	0,380	0,032	—1.158	1.032
4	0.841	0.362	0.392	0.030	—1.235	0.347
5	0.786	0,367	0.395	0,028	—1,317	0.153
6	0.731	0,371	0.398	0.027	—1.349	0.093
7	0.675	0.374	0.401	0,027	—1.356	0.070
8	0.621	0,377	0.404	0,027	—1.356	0,045
9	0.569	0.378	0.405	0,027	—1.361	0.027
10	0.516	0,378	0.406	0.028	—1.372	0.017
11	0.461	0,379	0.407	0.028	—1.381	0.020
12	0.405	0.380	0,408	0.028	—1,382	0.025
13	0.347	0.381	0,408	0.027	—1.355	0.025
14	0.289	0,382	0.409	0.027	—1.285	0.045
15	0,234	0.383	0.410	0.027	—1.171	0.080
16	0.185	0,388	0,416	0,028	—1.025	0.175
17	0.147	0,451	0.512	0.061	—0,871	2.438
18	0.119	0,542	0.598	0.056	—0,709	2.080
19	0.099	0,620	0,690	0.070	—0.567	1.485
20	0.083	0.659	0.755	0,096	—0.456	1,025
21	0.063	0.690	0.809	0,119	—0.377	0.650
22	0,055	0.713	0.843	0,130	—0,322	0.397
23	0.042	0,719	0.867	0.148	—0.277	0,228
24	0.031	0.724	0,899	0.175	—0,233	0.128
25	0.022	0.730	0.917	0.187	—0.189	0.108
26	0.015	0.733	0.924	0.191	—0.159	0.038
27	0.010	0.750	0.938	0.188	—0.124	1,438
28	0.007	0.795	0,955	0.160	—0.087	1.003
29	0.004	0.835	0.965	0.130	—0.063	0.882

The argument Z_a is a dimensionless number, since the temperature $t_0(N, \tau)$ and moisture content $u_v(\tau)$ are referred to their scale values and are dimensionless. The argument ζ_a has the dimension of time:

$$\zeta_a(N_0, N_1, \vartheta) = \frac{\Delta(N_0, N_1, \vartheta)}{u_v'(\vartheta)} = \frac{25}{3} \text{ hr}, \tag{52}$$

where the time ϑ is expressed in hours. The thermal diffusivity was calculated according to

$$a = 31.75 \cdot 10^{-6} \frac{\partial Z_a}{\partial \zeta_a} \text{ m}^2/\text{hr}.$$

It decreases as the moisture content decreases, and, at the end of the process, when the moisture breaks up into threads, and conductive heat transfer through the liquid is impossible, it is approximately six times less than at the beginning. The decrease in the thermal conductivity of the substance is even more notable:

$$\lambda = ac\gamma = a[358 + 436u]\ \mathrm{W/m \cdot {}^\circ C}$$

which is referred, as are the other transfer parameters, to the mean temperatures $\bar{T}$ and moisture contents $\bar{u}$ on the segment [0; 0.902].

Table 2

Arguments of the Characteristic Functions and Internal Transfer Parameters εKo and λ as Functions of Temperature and Moisture Content.

time, min	t, °C	u', $\frac{\text{kg water}}{\text{kg substance}}$	Z_a	ζ_a	Numerical expressions	λ, W/m · degree	ε Ko
20	32.0	0.251	−5.467	−0.1130	$\frac{\partial Z_a}{\partial \tau} = -17.21$	1.77	1.34
40	40.9	0.241	−2.200	−0.0377	$\frac{\partial \zeta_a}{\partial \tau} = -0.287$	1.72	0.06
60	43.7	0.229	−0.891	−0.0276		1.68	0.77
80	44.8	0.215	−0.281	−0.0243	$\frac{\partial Z_a}{\partial \zeta_a} = 60.0$	1.63	1.18
100	45.1	0.201	−0.116	−0.0213		1.57	1.15
120	45.4	0.187	−0.069	−0.0200	$a = 22.5 \cdot 10^{-4}$	1.51	1.13
140	45.6	0.173	−0.052	−0.0199		1.47	1.14
240	46.1	0.104	−0.0181	−0.0203	$\frac{\partial Z_a}{\partial \tau} = -12.89$	1.02	1.02
260	46.2	0.089	−0.0185	−0.0199	$\frac{\partial \zeta_a}{\partial \tau} = -0.253$	0.98	1.00
280	46.2	0.074	−0.0350	−0.0210		0.92	1.04
300	46.3	0.060	−0.0683	−0.0230	$\frac{\partial Z_a}{\partial \zeta_a} = 50.95$	0.87	1.11
320	46.7	0.047	−0.171	−0.0273		0.84	1.22
340	52.6	0.038	−2.800	−0.0702	$a = 19.1 \cdot 10^{-4}$	0.80	0.78
360	59.9	0.030	−2.930	−0.0790		0.78	1.10
540	80.0	0.003	−9.66	−1.516	$\frac{\partial Z_a}{\partial \tau} = -56.88$	0.13	4.52
560	82.8	0.002	−12.14	−1.839	$\frac{\partial \zeta_a}{\partial \tau} = -6.08$	0.13	5.07
580	85.2	0.001	−14.21	−2.003	$\frac{\partial Z_a}{\partial \zeta_a} = 9.35$ $a = 3.51 \cdot 10^{-4}$	0.13	4.53

The numbers Pn, Lu and the moisture conductivity of the substance are presented in Table 3. Calculation of these numbers is possible when the moisture content at one point of the body is known. It may be considered that from the beginning of the fall in drying rate (τ = 12), the moisture content of the body surface is zero. In this case the experimental quantity

$$y_p = \omega(N_0, N_1, 1, \tau) = -\frac{(k+2)}{2}(N_1^2 - N_0^2)\frac{\partial Z_a}{\partial \zeta_a} = u_v(\tau),$$

and the Z_p number is

$$Z_p = -\frac{(k+2)}{2}(N_1^2 - N_0^2)\frac{\partial Z_a}{\partial \zeta_a}\frac{u_v(\tau)}{u_v'(\tau)}.$$

In Table 3 it has been calculated from the formula

$$Z_p(0;\ 0.902;\ 1;\ \tau) = -1.22\frac{\partial Z_a}{\partial \zeta_a}\frac{u_v(\tau)}{u_v'(\tau)}.$$

The simplex Lu^{-1} has been computed as a characteristic function of the thermodynamics of irreversible processes (45), and its arguments Z_p and ζ_a have been evaluated as functions of time. The sign of the thermal diffusion number is explained by the difference in writing the transfer equation system in [1] and [11, 12]. Its value decreases sharply when the moisture breaks up into threads.

The inverse problem of external heat and mass transfer may be solved from the internal parameters and the gradients of the reconstructed fields on the body surface.

Table 3

Values of the Argument Z_p and of the Parameters Pn, Lu and a_m

Time, min.	Z_p	Numerical expressions	Lu	$a_m \cdot 10^4$
240	18.66	$\frac{\partial Z_p}{\partial \tau} = -32.04$	0.0471	0.90
260	16,32		0.0530	1.01
280	13.77	Pn = −2.36	0.0609	1.16
300	12.33		0,0656	1.25
320	11.30		0.0677	1.29
340	10.95		0.0504	0.964
360	11.10		0.0474	0.905
540	5.10	$\frac{\partial Z_p}{\partial \tau} = -12.89$	00.120	0.422
560	4.84	Pn = −0.227	0.114	0.401
580	4.07		0.120	0.422

Relation between the thermal diffusivity and the phase transition criterion. The diffusivity was calculated from (34), in which R is a characteristic dimension of the body, N and N_0 are dimensionless coordinates of the points of temperature measurement, and Z_a and ζ_a are arguments of characteristic functions of Massieu-Gibbs type in the thermodynamics of irreversible processes, which depend appreciably on time.

In the calculations it is convenient to determine the number A, which is proportional to the diffusivity

$$A = 2k\,[N^2 - N_0^2]^{-1}\, a\, \vartheta_0\, R^{-2}. \tag{53}$$

It coincides with the Fourier number to within a multiplying factor. Instead of the argument ζ_a, which has the dimension of time, it is convenient to introduce its dimensionless variant

$$\zeta = \zeta_a/\vartheta_0, \tag{54}$$

divided by the time scale ϑ_0. Clearly, in this notation, the dimensionless diffusivity is

$$A = \partial Z/\partial\zeta; \qquad Z = Z_a, \tag{55}$$

and the Kossovich number is

$$\varepsilon\,\mathrm{Ko} = Z - \zeta A. \tag{56}$$

It follows from (54) that the variation of argument Z of the thermodynamic functions ε Ko and A may be represented by the integral

$$Z - Z_0 = \int_{\zeta_0}^{\zeta} A\, d\zeta, \tag{57}$$

and the variation of the second argument of these characteristic functions may also be expressed in terms of the dimensionless diffusivity A and the other argument Z

$$\zeta - \zeta_0 = \int_{Z_0}^{Z} A^{-1}\, dZ. \tag{58}$$

Substitution of (57) is the foregoing equality leads to the expression

$$\varepsilon\,\mathrm{Ko} = Z_0 + \int_{\zeta_0}^{\zeta} A\, d\zeta - \zeta A, \tag{59}$$

by which the complex number $\varepsilon\,\mathrm{Ko}$ is represented in the form of a function of number A and of the other dimensionless argument ζ, both of which may be determined experimentally. If (58) is substituted in (56), the complex $\varepsilon\,\mathrm{Ko}$ will be represented as a function of the diffusivity and the argument Z

$$\varepsilon\,\mathrm{Ko} = Z - A\left[\zeta_0 + \int_{Z_0}^{Z} A^{-1} dZ\right]. \tag{60}$$

The arguments of the characteristic functions may be found from (55) and (56) as functions of the number $\varepsilon\,\mathrm{Ko}$. In particular,

$$Z = \zeta Z_0 \zeta_0^{-1} - \zeta \int_{\zeta_0}^{\zeta} \varepsilon\,\mathrm{Ko}\,\zeta^{-2} d\zeta, \tag{61}$$

$$\frac{\zeta}{\zeta_0} = \exp\left[\int_{Z_0}^{Z} (Z - \varepsilon\,\mathrm{Ko})^{-1} dZ\right]. \tag{62}$$

Hence the dimensionless thermal diffusivity may be represented as a function of the number $\varepsilon\,\mathrm{Ko}$:

$$A = Z_0 \zeta_0^{-1} - \varepsilon\,\mathrm{Ko}\,\zeta^{-1} - \int_{\zeta_0}^{\zeta} \varepsilon\,\mathrm{Ko}\,\zeta^{-2} d\zeta, \tag{63}$$

$$A = (Z - \varepsilon\,\mathrm{Ko})\,\zeta_0^{-1} \exp\left[-\int_{Z_0}^{Z} (Z - \varepsilon\,\mathrm{Ko})^{-1} dZ\right]. \tag{64}$$

The integral type relations (58), (59), (63), (64) allow us to calculate one transfer parameter by integration from the other experimentally determined parameters. Of course, these calculations are possible when the arguments of the Massieu-Gibbs type characteristic functions are known from experiment as a function of time. All these integral relations derive from the differential relations

$$\frac{\partial\,\varepsilon\,\mathrm{Ko}}{\partial Z} = -\frac{\partial \ln A}{\partial \ln \zeta}, \tag{65}$$

$$\frac{\partial\,\varepsilon\,\mathrm{Ko}}{\partial \ln \zeta} = -\frac{1}{2}\frac{\partial A^2}{\partial Z}, \tag{66}$$

which are equivalent to their definitions (55) and (56).

Relation between thermal diffusion and inertia numbers. The thermal diffusion and inertia numbers

$$\mathrm{Pn} = -\frac{\partial \Pi}{\partial Z}; \quad \mathrm{Lu}^{-1} = \Pi - \zeta \frac{\partial \Pi}{\partial \zeta} \quad (\Pi = Z_p) \tag{67}$$

are interrelated by the somewhat different differential relations of the thermodynamics of irreversible processes

$$\frac{\partial\,\mathrm{Lu}^{-1}}{\partial Z} = \frac{\partial\,\mathrm{Pn}}{\partial \ln \zeta}, \tag{68}$$

$$\frac{\partial\,\mathrm{Lu}^{-1}}{\partial \ln \zeta} = A^2 \frac{\partial\,\mathrm{Pn}}{\partial Z}, \tag{69}$$

which is due to the different role of the time-dependent arguments Z, ζ and Π in the definitions (67). However, the integral formulas, which permit calculations of one transfer parameter from the other, hold in this case also. It follows from the definitions (67) that

$$Z - Z_0 = \int_{\Pi}^{\Pi_0} \mathrm{Pn}^{-1} d\Pi; \quad \Pi - \Pi_0 = \int_{Z}^{Z_0} \mathrm{Pn}\, dZ; \tag{70}$$

$$\frac{\Pi}{\zeta} - \frac{\Pi_0}{\zeta_0} = -\int_{\zeta_0}^{\zeta} \mathrm{Lu}^{-1} \zeta^{-2} d\zeta; \tag{71}$$

$$\zeta = \zeta_0 \exp \int_{\Pi_0}^{\Pi} (\Pi - \mathrm{Lu}^{-1})^{-1} d\Pi. \tag{72}$$

The inertia number may be represented as a function of the thermal diffusion number and the parameters of arguments Z and ζ

$$\mathrm{Lu}^{-1} = \Pi_0 + \zeta \frac{\partial Z}{\partial \zeta} \mathrm{Pn} - \int_{Z_0}^{Z} \mathrm{Pn}\, dZ, \tag{73}$$

if (70) is substituted into the definition of Lu^{-1} (67). It is more important to express the thermal diffusion number in terms of the inertia number and the same arguments Z and ζ. Substitution of (71) into the definition of Pn (67) leads to the relation

$$\mathrm{Pn} = \frac{\partial \zeta}{\partial Z} \left[\int_{\zeta_0}^{\zeta} \mathrm{Lu}^{-1} \zeta^{-2} d\zeta + \mathrm{Lu}^{-1} \zeta^{-1} - \Pi_0 \zeta_0^{-1} \right] . \tag{74}$$

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